11/5/19

M159 (Continued)

**Remarks:**

1) $\Pr(J=j \mid T_x = t) = \dfrac{\mu_{x+t}^{(j)}}{\mu_{x+t}^{(\tau)}}$

2) $\Pr(J=j \mid T_x \le n) = \cancel{{}_n q_x} \; \dfrac{{}_n q_x^{(j)}}{{}_n q_x^{(\tau)}}$

## Standard Service Table Notation

$l_x = l_x^{(\tau)}$ ($l_x^{(j)}$ is never used)

${}_n d_x^{(j)}$ = # of $x$-year olds who depart by decrement $j$ within $n$ years

$${}_n d_x^{(\tau)} = \sum_j {}_n d_x^{(j)}$$

Examples: See P.S of LTAM Tables

1) $q_{35}^{(i)} = \dfrac{d_{35}^{(i)}}{l_{35}} = \dfrac{i_{35}}{l_{35}} = \dfrac{213.3}{218833.9}$

2) ${}_3 q_{40}^{(w)} = \dfrac{{}_3 d_{40}^{(w)}}{l_{40}} = \dfrac{\cancel{d_{40}}\, w_{40} + w_{41} + w_{42}}{l_{40}}$

3) ${}_{10|2} q_{51}^{(r)} = \dfrac{{}_2 d_{61}^{(r)}}{l_{51}} = \dfrac{r_{61} + r_{62}}{l_{51}}$

Associated Single Decrement Table (ASDT)
(Absolute Rates of Decrement)
(Primed Probabilities)

Idea: In the multiple decrement context, close all doors except the one under consideration.
(Back to single-life & single-decrement.)

Notation: ${}_n d_x'^{(j)}$ = the # of x-year olds who depart by door j within the next n years, given that door j is the only door open

$$\frac{{}_n d_x^{(j)}}{l_x} \leq \frac{{}_n d_x'^{(j)}}{l_x}$$

$${}_n q_x^{(j)} \leq {}_n q_x'^{(j)}$$

${}_n p_x^{(j)}$ is never used

${}_n p_x'^{(j)}$ is used (same as old ${}_n p_x$)

Remarks: 1) When we say "survive a decrement", we mean in the ASDT context.

2) $\mu_{x+t}'^{(j)} = \mu_{x+t}^{(j)}$ (the prime notation is not used on the $\mu$'s)

3) $\mu_{x+t}^{(\tau)} = \sum_j \mu_{x+t}^{(j)}$

$${}_n p_x^{\prime(j)} = e^{-\int_0^n \mu_{x+t}^{(j)}\,dt}$$

$${}_n p_x^{(\tau)} = e^{-\int_0^n \mu_{x+t}^{(\tau)}\,dt} = e^{-\int_0^n \left(\sum_j \mu_{x+t}^{(j)}\right)dt}$$

$$= \prod_j e^{-\int_0^n \mu_{x+t}^{(j)}\,dt}$$

$$\therefore \quad {}_n p_x^{(\tau)} = \prod_j {}_n p_x^{\prime(j)}$$

Then ${}_n q_x^{(\tau)} = 1 - {}_n p_x^{(\tau)}$